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# THE CONVERSION OF FUNDAMENTAL TRACKING DATA TO METRIC FORM

P. E. SCHMID

JANUARY 1969



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P. E. Schmid

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Goddard Space Flight Center  
Greenbelt, Maryland

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## THE CONVERSION OF FUNDAMENTAL TRACKING DATA TO METRIC FORM

### ABSTRACT

The fundamental tracking data recorded during the track of a space vehicle by a ground station seldom directly expresses the physical measurements involved. In order to directly interpret measured tracking data in terms of physical phenomena, prior to orbit or trajectory computation, a conversion of fundamental tracking data to metric form is required. This conversion is not trivial since it requires an exact knowledge of how each measurement is made, a factor which is a function of both tracking system type as well as system configuration during a particular satellite pass. The tracking data conversion capability discussed in this report was implemented to provide a straight forward means of converting various tracking data types to metric form (i.e. range, range-rate and angles) free from any data preprocessing such as mathematical smoothing or statistical editing. Such metric data is then used as basic information in post-flight tracking data analysis.

# THE CONVERSION OF FUNDAMENTAL TRACKING DATA TO METRIC FORM

## 1.0 PURPOSE

The purpose of this report is to document the tracking data conversion capability which has been implemented by the Mission and Systems Analysis Branch of the Goddard Space Flight Center (GSFC). Included is a detailed discussion of the meaning of each of the metric quantities and associated "time tags" appearing in the standardized output magnetic tape and printout. This standardized format has been implemented in order to facilitate the various post-flight orbital and tracking network analyses now being conducted by the Mission and Trajectory Analysis Division (MTAD) of the Goddard Space Flight Center (GSFC).

## 2.0 INTRODUCTION

The primary objective of this programming effort was to provide a straightforward means of converting various tracking data types to metric form, free from such data preprocessing as mathematical smoothing or statistical editing. Metric data is defined in this report as measurements of range in kilometers, range-rate in kilometers per second and angles in degrees with azimuth relative to true North. This is in contrast to the basic information generally contained in the raw tracking data – for example, Doppler phase counts, range time delays, angles in octal format and so on. The metric output is useful to both engineers associated with tracking system and subsystem analysis as well as to trajectory analysis personnel.

The Tracking Data Editing System (TRADES) program and the related autocorrelation and automatic plotting analysis programs indicated in Figure 1 have been written specifically for the recently installed IBM 360-95 computer assigned to the Tracking and Data Systems Directorate of which MTAD is an integral part. The Metric Calibration and Formatting (METCAF) program converts Unified S-Band and C-Band Radar data. The STADAN program converts Goddard Range and Range-Rate (GRARR) data. The latter program includes an integrated trajectory routine which results in the resolution of range ambiguity. A planned addition is the conversion of MINITRACK data to the standardized output format. MINITRACK is a NASA tracking scheme which operates at 136 MHz providing angle data only. The TRADES 360-95 program incorporates the flexibility needed to

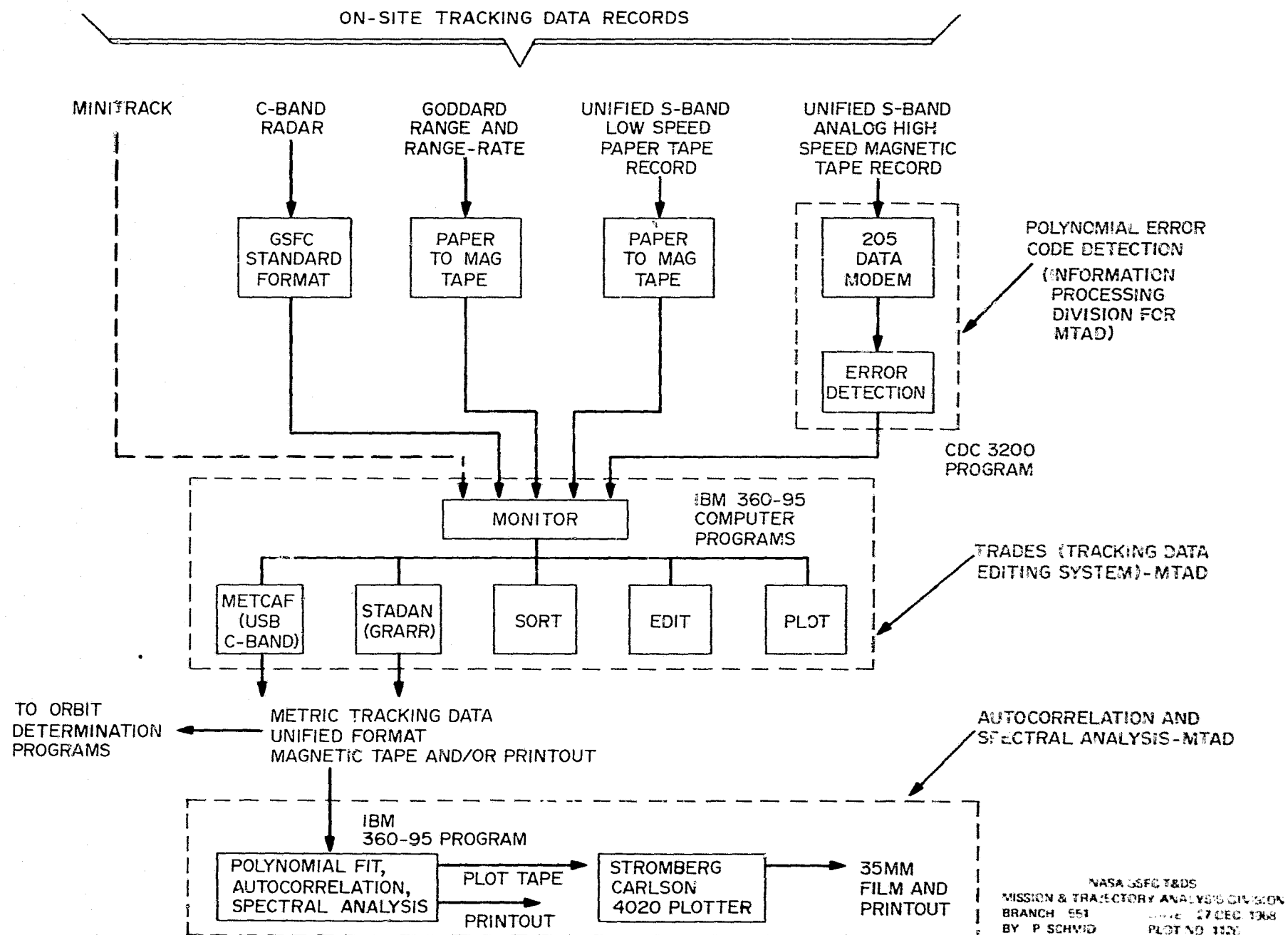


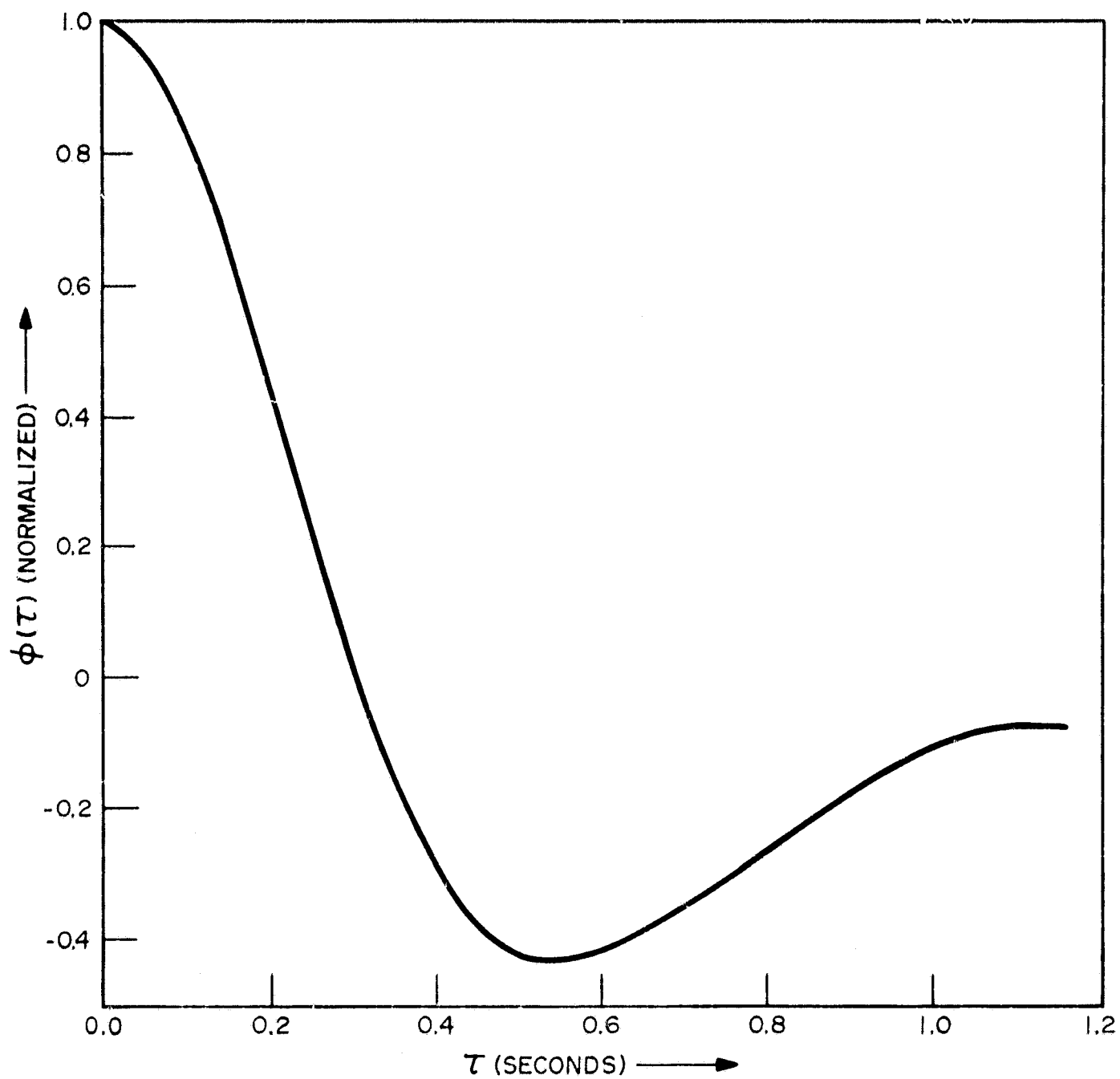
Figure 1-Tracking Data Conversion and Analysis

easily insert pertinent scalar constants or metric computational procedure as required for any given post-flight raw tracking data conversion. Subroutines of TRADES permit selection of up to 500 data time arcs per given input tape (ref. 1).

By keeping the tracking data conversion free from preprocessing, meaningful autocorrelation and spectral noise analyses are being conducted to provide a measure of on-site equipment performance. As shown in figure 1, the analysis programs are entirely independent of the basic metric data conversion except that the metric output is used as the analysis input. This technique permits a degree of experimental freedom in determining the extent to which automated tracking data analysis can be efficiently employed prior to postflight orbital analysis. Figure 2 is an example of what can be done with such techniques. This figure shows the normalized autocorrelation function of angle noise derived from a stretch of X-angle tracking data acquired by the Apollo Carnarvon, Australia Tracking Station (CRO) during the Apollo AS-501 mission. This curve was automatically plotted by means of the Stromberg Carlson 4020 computer plotter. Closed loop angle servo bandwidth and transfer functions are readily derived from such curves. The Fourier transform related noise power spectral density curves are also plotted automatically. In this manner the tracking system performance is ascertained from data corresponding to an actual vehicle track rather than from the usual simulated data performance tests (ref. 2). The subject of fundamental tracking data analysis will be discussed in a subsequent report.

### 3.0 THE MEANING OF METRIC DATA

The so-called "raw tracking data" as recorded during a test or mission either at a particular tracking site or at a NASA facility such as Goddard is seldom in a form which is readily interpreted in terms of the quantities of interest, namely, range, average range-rate, angles and associated timing. For example, "raw data" is sometimes recorded in a base eight numbering system to facilitate real time orbit computation, Doppler cycle counts contain an inserted constant bias count to permit determination of the direction of radial range change, WWV timing signal radiowave propagation delay may not be included in the associated time tag and so on. In addition, the raw data is closely linked to hardware configuration and operational procedure, areas not always familiar to persons concerned with trajectory analysis. Since all NASA tracking systems are in a continual state of evolution it is important that this postflight conversion from raw data to metric data be performed in a manner which properly reflects all changes applicable to any given data stretch. The purpose of the TRADES 360-95 program is to provide the basic metric output associated with the various types of raw tracking data available for post-flight analysis.



APOLLO AS-501 9 NOVEMBER 1967  
 CRO (CARNARVON, AUSTRALIA)  
 $\sigma_x = 0.052$  MILLIRADIANS  
 MISSION AND TRAJECTORY  
 ANALYSIS DIVISION, GSFC

NASA-GSFC-T&DS  
 MISSION & TRAJECTORY ANALYSIS DIVISION  
 BRANCH 551 DATE 27 DEC. 1968  
 BY P. SCHMID PLOT NO. 1089

Figure 2--Autocorrelation Analysis of X-Angle Residuals

This basic metric information consists of one or more of the following data types:

1. angles (based upon angle of arrival of signals at the tracker)
2. radial range (based upon measured time delays)
3. radial range change (based upon measured phase changes -- usually termed "Doppler frequency" measurements).

In addition, each of these data types is associated with a particular "time tag." In order to properly interpret the metric data one must be aware of the exact method employed in obtaining the metric information as well as all bias corrections and all computational equations applied either on-site or during any data pre-processing. No data smoothing is employed in the TRADES program, however selective editing and time sorting is available. The editing is based upon station identification (ID) codes, sometimes referred to as data "flags." These include indications of tracking station name, tracked vehicle type, "autotrack," "valid range" and "valid range-rate." Three basic types of tracking systems are currently handled by TRADES -- namely, Unified S-Band (USB), Goddard Range and Range-Rate (GRARR) and C-Band pulse radar (FPQ-6 and FPS-16) (ref. 3, 4, 5). The output metric format is independent of data sampling rate. The sampling rate, however, will influence the users data handling procedure or interpretation especially when using average range-rate information or whenever performing autocorrelation analyses. As will be shown, there is a strong similarity between USB and GRARR Doppler data handling. This is expected since the basic measurement in both cases is that of radial change of range of a vehicle relative to the ground tracking station based upon a measurement of radio frequency carrier Doppler cycle change. The following paragraphs will describe in detail the significance of the TRADES output format (figure 3) for the USB, GRARR and C-Band tracking systems.

#### 4.0 UNIFIED S-BAND (USB) SYSTEM

The Unified S-Band (USB) system incorporates a coherent transponder at the vehicle being tracked. The up-link nominal 2 GHz signal is multiplied in frequency by a constant ( $k = 240/221$ ) and then retransmitted to the interrogating station. Such coherent ground Doppler tracking is termed 2-way Doppler tracking. If the interrogating signal is absent or drops below a certain level (-128 dBm, ref. 6) an auxiliary oscillator in the transponder is switched in and subsequent ground Doppler tracking of such a signal is termed one-way Doppler tracking. If one station transmits a carrier signal to the spacecraft and a second station receives the "turned around" signal (figure 4) the process is termed 3-way Doppler tracking

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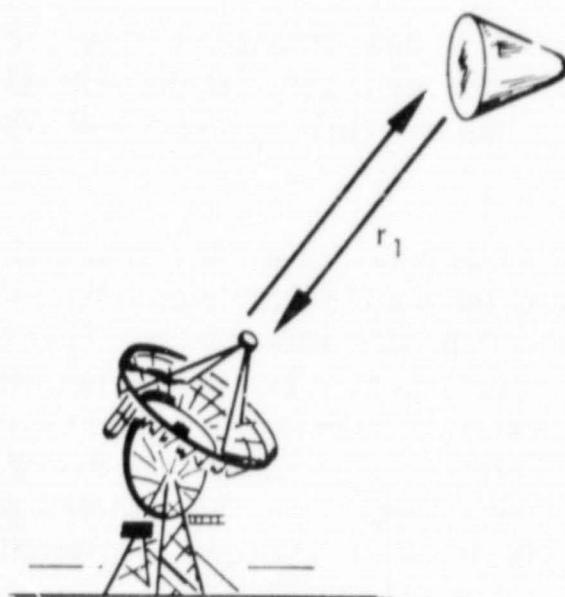
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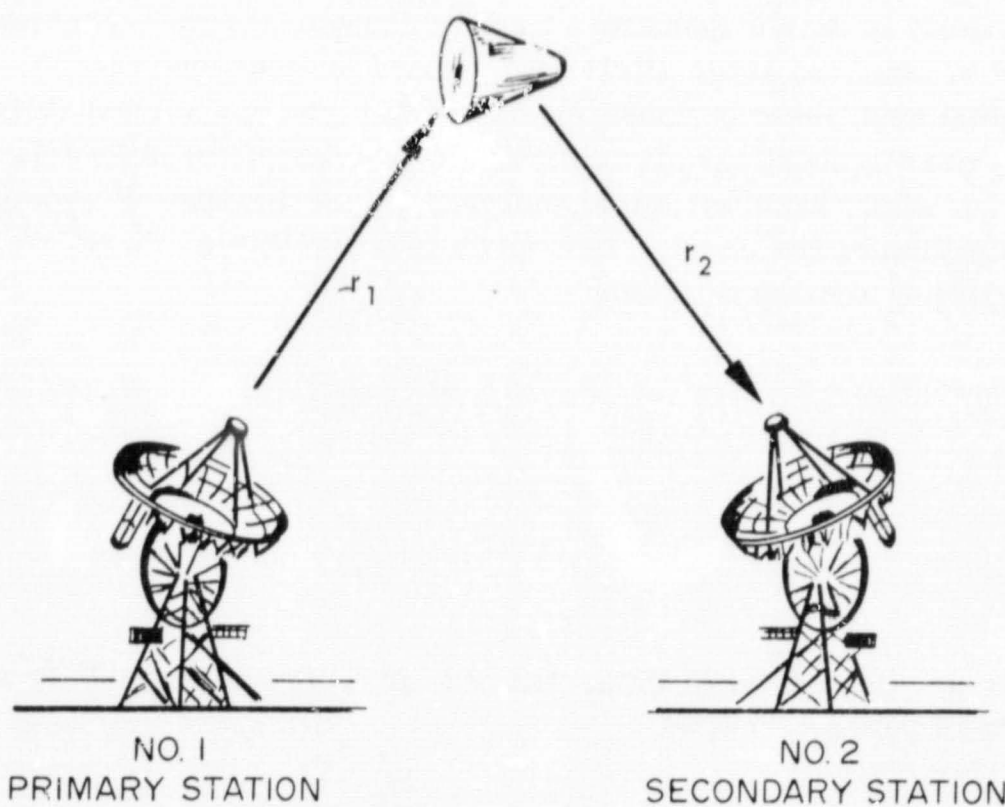
# UNIFIED FORMAT

DATA EXPLANATION	FORMAT	CARD COLUMN
STATION NUMBER	I2	1-2
DATA CONDITION CODES S-BAND ONLY	5I1	3-7
DATA CONDITION CODE C-BAND ONLY	I1	3
SAT ID NUMBER STADAN ONLY	I3	5-7
DAY OF YEAR	I3	8-10
HOUR	I2	11-12
MINUTE	I2	13-14
SECOND SS.FFFFFFF	F9.6	15-23
X-ANGLE +OR- 90 (DEG)	F7.3	24-30
Y-ANGLE +OR- 90 (DEG)	F7.3	31-37
AZIMUTH 0-360 (DEG)	F7.3	38-44
ELEVATION 0-90 (DEG)	F6.3	45-50
SLNT RANGE (KM)	F12.5	51-62
RANGE RATE (KM/SEC) OR DELTA RANGE	F9.6	63-71
SYSTEM INDICATOR	I1	72

Figure 3-TRADES Output Format



(A) TWO-WAY DOPPLER



(B) THREE-WAY DOPPLER

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BRANCH 551 DATE 27 DEC. 1968  
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Figure 4—Two-way and Three-way Doppler

(ref. 7). At present the metric data conversion is only concerned with 2-way coherent tracking, which is the most precise and normal mode of USB operation. However, the other modes can be easily incorporated into TRADES if deemed useful.

The USBS range measurement is made by means of an autocorrelation involving a pseudo random Code which is modulated onto the S-Band uplink carrier and coherently turned around by the transponder. That is, the locally generated code at the ground station undergoes a variable delay while being compared to the received code which has undergone a two-way radiowave propagation delay. When the inserted ground station delay equals the 2-way propagation delay the autocorrelation has a maximum value and the inserted ground time delay is a measure of slant range. With the "long code" or normal Pseudorandom noise code, the USB range measurement is unambiguous to a range of 800,000 km (ref. 8). Normally only one such "range acquisition" is made per pass over a given tracking station and subsequent range readouts are obtained by updating the initial measurement by integrating a "clock Doppler" signal. That is, once range acquisition is made, the ranging code is switched off and a clock modulation is switched on. The relative phase change of the clock signal as relayed via the spacecraft is then a measure of range change. As presently configured, the clock is not an integral submultiple of the carrier frequency (ref. 9, 10) however, the smallest increment of range change in the tracking format (termed the "range unit") corresponds to approximately 16 cycles of 2-way carrier Doppler change. That is, whenever the vehicle moves a radial distance of approximately 16 half-wavelengths at the carrier frequency relative to the ground station one range unit (RU) is recorded. Since

$$f \doteq 2.2 \text{ GHz}$$

and

$$\lambda = \frac{c}{f}$$

where  $C$  = the speed of light. One Range Unit corresponds to approximately one meter of range. That is,

$$(16) \left( \frac{\lambda}{2} \right) = 1.09 \text{ meters}$$

It should be noted that the range update is done at the site and from an equipment standpoint is essentially independent of the carrier Doppler tracking

information which is also contained in the raw USB data format. Only the receiver radio frequency and intermediate frequency stages are common to the range and range-rate channels.

The USB dish antennas employ an X-Y mount. The 30-foot diameter antennas employ an X-axis aligned North-South whereas the 85 foot X-axis is aligned East-West. The X-axis is always contained in the local earth tangent plane.

#### 4.1 USB RANGING

The basic range information in the raw USB data is the range unit. Originally on-site corrections were included for radiofrequency time delays, such as those attributable to waveguide runs and transponder time delay. Beginning with Apollo AS-205 the transponder delay was not inserted at the tracking site and must be subtracted by the user (ref. 6). This is done within METCAF such that the range output in kilometers represents the best available estimate of measured range prior to any statistical editing. The raw data time tag associated with the range corresponds to UTC (Universal Time Coordinated, ref. 11) ground receive time and includes an on-site correction for WWV radiowave propagation time delay. The equation which relates range units to range (ref. 10) is given by:

$$R = k \text{ (RU)} (10^{-3}) \text{ kilometers, where RU = range units as in the on-site (1) recorded format and;}$$

$$k = \frac{c}{576 f_{\text{clock}}}$$

$$c = \text{speed of light} = 2.997925 \times 10^8 \text{ m/s (ref. 7)}$$

$$f_{\text{clock}} = (495.8333) (10^3) \text{ Hz}$$

These constants are appropriate starting with Apollo Mission AS-502 (launched April 4, 1968). Prior to this time the range granularity, k, for Apollo missions AS-501 and AS-204L was slightly different since different clock frequencies were employed. The appropriate values are listed in Table I.

Table I  
USB Range Granularity Used by GSFC

Mission	Apollo Number	Launch Date	Range Granularity (k) m/range unit
AS-501	4	Nov. 9, 1967	*1.0493409
AS-204L	5	Jan. 22, 1968	1.0493409
AS-502	6	April 4, 1968	1.0496936

\*Except for the Canary Island Station (CYI) where  $k = 1.0484546$  m/range unit.

The output of METCAF for USB ranging is thus radial range in kilometers corrected for on-site time delays and transponder time delay. The time tag is ground receive time corrected for WWV propagation delay. An updated value of range is inserted into the basic tracking message at least once every six seconds. One frame per six seconds corresponds to the normal "low speed" data rate. This low speed data is derived on-site from the high speed data. The high-speed data is simultaneously available at a rate of 10 frames per second, 5 frames per second or 2.5 frames per second depending on operator selection at the on-site USB "Tracking Data Processor." Each high speed raw data frame consists of 240 bits. Generally the lower rates are used by outlying stations which are constrained to using lower data rates via the NASA Communications (NASCOM) network which relays all tracking data to the Goddard Space Flight Center and to the Manned Spacecraft Center. It should be noted that USB sites often obtain carrier Doppler and angle data without ranging. This is in contrast to the Goddard Range and Range-Rate system which always provides range data.

## 4.2 USB Range-Rate

The basic measurement of range-rate in the USB system is that of carrier Doppler phase change. That is, the down link carrier from the spacecraft is coherently tracked by a phase-locked ground receiver (ref. 12) as indicated in figure 5. In 2 and 3-way Doppler, as defined in section 4.0, the essential system functions are:

1. Up-link fixed frequency nominal 2 GHz carrier transmission, frequency derived from Rubidium Clock source, stability 1 part in  $10^{11}$ .
2. Transponder (ref. 13) receiver aboard spacecraft phase-locked to up-link frequency plus up-link Doppler shift.

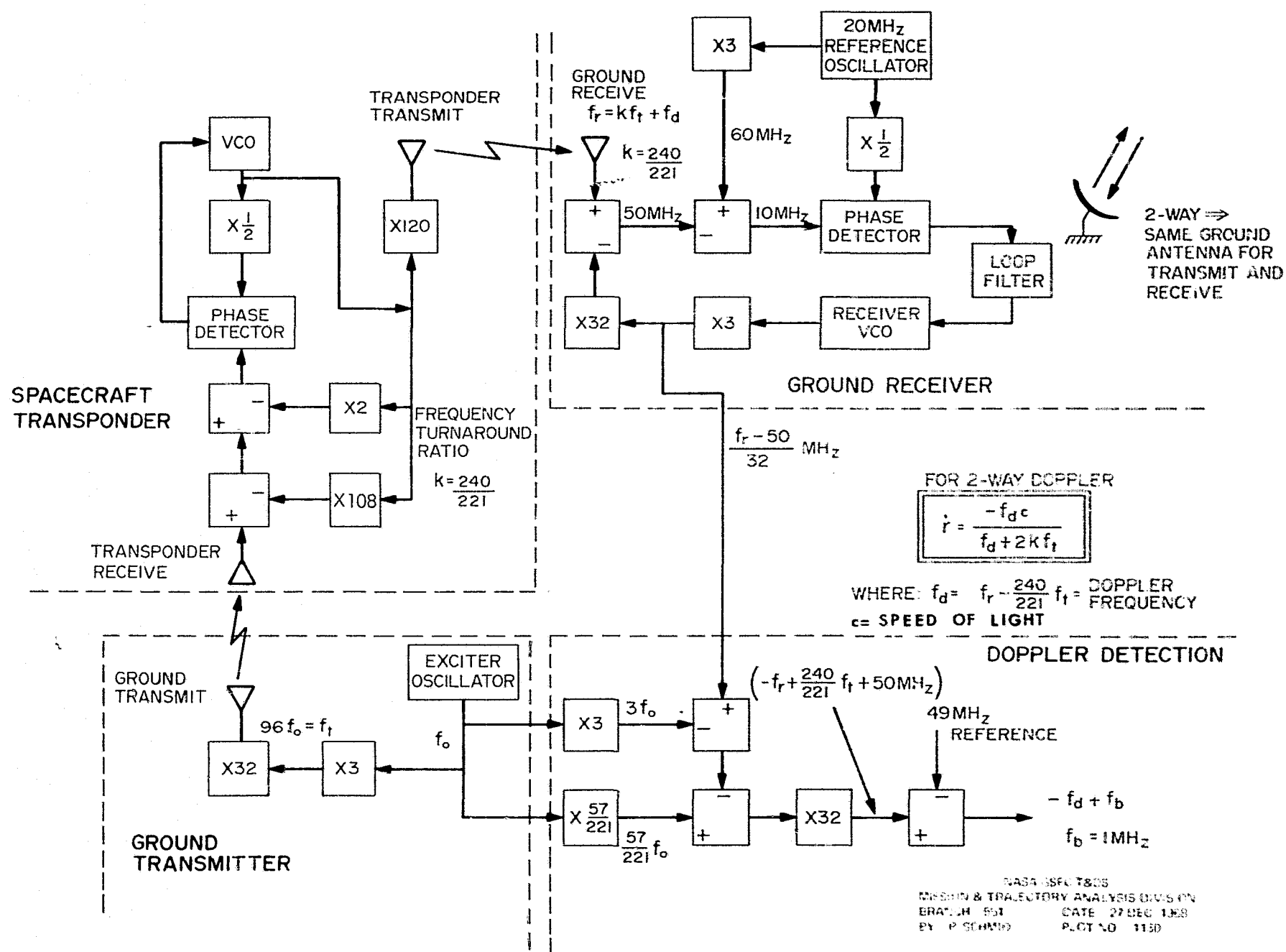


Figure 5—Unified S-Band Doppler Tracking

3. Transponder transmitter frequency coherently derived from up-link carrier plus up-link Doppler shift. Turnaround ratio fixed at 240/221 for all USB tracking.
4. Ground receiver is phase locked to down-link signal which is at the transponder output frequency plus the down-link Doppler frequency shift.
5. The ground receiver signal is compared to a signal coherent with the transmitted carrier in such a manner that the basic output, as indicated in figure 5, is minus Doppler frequency plus a stable 1 MHz bias. The minus sign in this case means that a decreasing radial range (i.e. negative) produces a Doppler plus bias frequency less than 1 MHz and vice versa. The 1 MHz bias is derived from the Rubidium frequency standard.

It should be noted that rather than a "Doppler frequency" the raw data consists of a cycle count of phase change which in the one-way or two-way modes of operation is a direct measure of spacecraft radial range change relative to a given ground tracking station. For two-way Doppler the average radial range-rate is given by:

$$\dot{r}_a = \frac{\left(\frac{\Delta N}{\Delta T} - f_b\right) c}{-\left(\frac{\Delta N}{\Delta T} - f_b\right) + 2k f_t} = \frac{-f_d c}{f_d + 2k f_t} \text{ km/s} \quad (2)$$

where:

$\dot{r}_a$  = average range-rate in vehicle time

$c = 2.997925 \times 10^5 \text{ km/s}$  (speed of light)

$f_b$  = inserted bias =  $10^6 \text{ Hz}$

$k = \frac{240}{221}$  turnaround ratio built into all USB transponders

$f_t$  = up-link frequency =  $96 f_0$

$f_0$  = basic oscillator frequency

$f_d$  = average Doppler frequency shift

$\Delta N$  = cycles of phase change

$\Delta T$  = time period over which phase change was observed.

The frequency  $f_0$  is the basic exciter frequency indicated in figure 5 and for the Apollo Lunar Module Interrogation, is equal to  $21.893772 \times 10^6$  Hz. For the Apollo Command Module Interrogation  $f_0$  is equal to  $21.941732 \times 10^6$  Hz. For Lunar Orbiter or other post-flight analysis  $f_0$  will be different and the value must be obtained from post-flight tracking logs. Equation 2, which is correct in terms of special relativistic effects, is derived from phase considerations in the Appendix.

It is again emphasized that the basic measurement is a cycle count of carrier phase shift plus bias  $\Delta N$  over a time period  $\Delta T$ . If the time interval  $\Delta T$  is fixed for each measurement the scheme is termed "constant T count." If the cycle accumulation  $\Delta N$  is fixed for each measurement the scheme is termed "Constant N count."

#### NON-DESTRUCT DOPPLER

The normal mode of USB Doppler tracking is constant T count non-destructive. The term non-destructive refers to the fact that whereas the counter is read-out at even time intervals the accumulated count is not destroyed. Thus the average frequency is obtained in this case by differencing the count appearing in adjacent frames and dividing by the sample time.

That is,

$$\frac{\Delta N}{\Delta T} = \frac{n_2 - n_1}{T} = -f_d + 10^6 \text{ Hz} \quad (3)$$

where

$f_d$  = average Doppler frequency shift

$n_2$  = raw data count at a given frame time

$n_1$  = raw data count at previous frame

$T$  = time interval between frames

In the METCAF program the sample time  $T$  is the actual difference in time between frames, which, because of possible data gaps is not necessarily equal to the frame-rate. For reasons elaborated upon in the Appendix the time tag associated with the average range-rate is taken mid-way between the frame sample interval. This time, which is UTC ground measurement time corrected for WWV radiowave propagation delay, must be corrected by the user to relate the time tag to vehicle time.

The USB destruct Doppler can be processed at any sample rate lower than the maximum raw data rate. That is, 10 per second data can be differenced every other frame, every tenth frame and so on as required.

## DESTRUCT DOPPLER

The USB System can also operate in the  $N$  count mode which means the information contained in the raw data frame is a measure of the time it took to accumulate a fixed number of Doppler plus bias cycles of phase change (ref. 14). There are two USB accumulated counts:

$$N_1 = 77824$$

$$N_2 = 778240$$

With this scheme the time required to count  $N_1$  or  $N_2$  cycles is recorded in terms of cycles from a 100 MHz timing oscillator. Thus for zero Doppler (i.e. a stationary vehicle), the count is due just to bias and in the  $N_1$  mode 7782400 cycles of 100 MHz would be recorded in the tracking data format. The time resolution is  $1/10^8$  or 10 nanoseconds. So for this case the average Doppler is related by;

$$\frac{\Delta N}{\Delta T} = \frac{N_1}{\Delta T} = \frac{77824}{m(10^{-8})} = -f_d + 10^6 \text{ Hz} \quad (4)$$

$f_d$  = average Doppler frequency shift

$m$  = cycles of 100 MHz recorded in data frame

for  $N_2$  count  $\Delta N = N_2 = 778240$ .

The time tag in METCAF for N count destruct USB Doppler data is at the mid-point of the count time interval as computed from the raw data time interval. This time tag includes the on-site WWV propagation delay correction. It is not possible to calculate the time required to accumulate a given number of Doppler cycles and then send out in real time this information time tagged to the beginning of the Doppler count.

In the MSFN USB system, which performs real time orbit computation, the destruct Doppler time tag lags by one data frame. That is, Doppler count time (Destruct Doppler only) on any given frame must be associated with the previous frame. This is accomplished by subtracting the interval of time between frames from the frame time tag. Again this is achieved within the METCAF program and does not concern the user. It should be pointed out that all normal real-time MSFNApollo orbit computation is achieved with non-destruct Doppler data.

#### 4.3 USB Angles

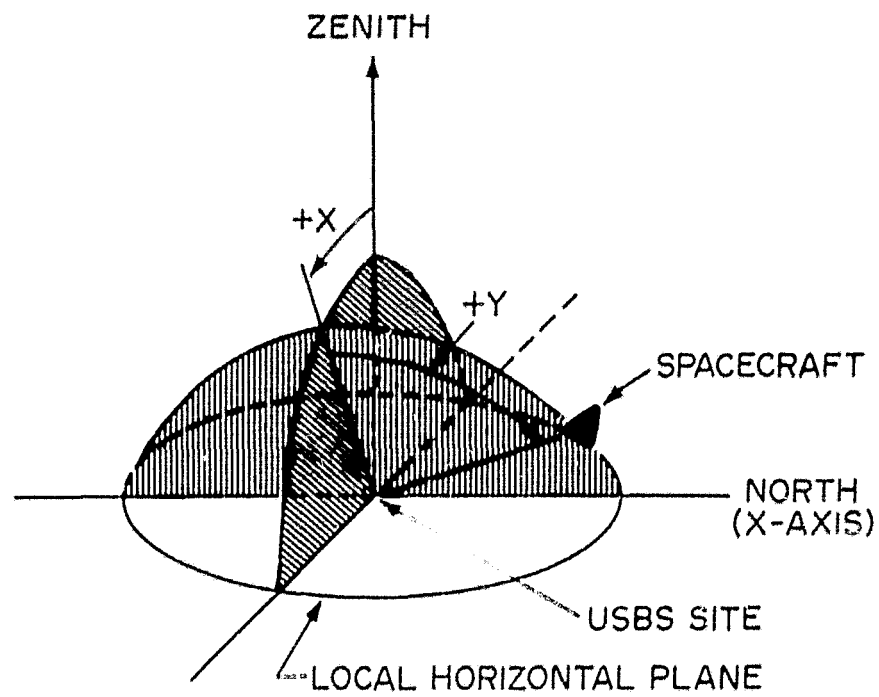
The basic measurement is x and y angles (ref. 7,15). The 85 foot sites have the X axis aligned East-West, while X-axis for 30 foot sites is aligned North-South (fig. 6). This angle information is contained in decimal degrees at the METCAF output. In addition, all angle data is converted to azimuth and elevation data and included in the METCAF output. This is a straight geometric computation where azimuth refers to angle measured from true North (0 to 360°) in a clockwise sense and elevation (0 to 90°) with zenith corresponding to 90°. These conversions are given by equations 5 through 10.

30 ft. USB antenna sites:

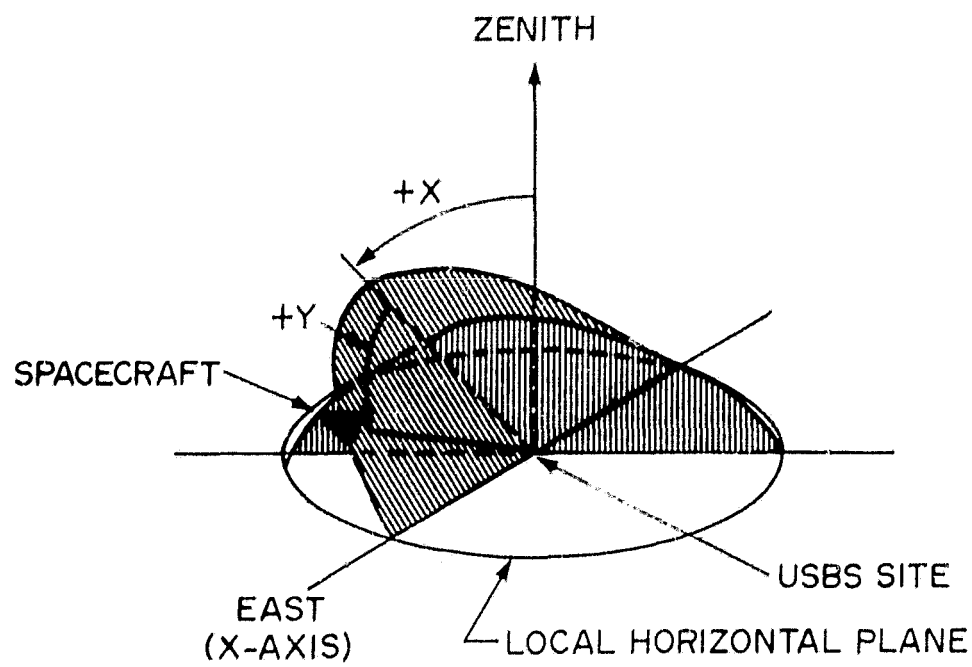
$$\sin A = \frac{\cos y \sin x}{\cos E} \quad (5)$$

$$\cos A = \frac{\sin y}{\cos E} \quad (6)$$

$$\sin E = \cos y \cos x \quad (7)$$



(A) 30-FT ANTENNA



(B) 85-FT ANTENNA

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Figure 6—Angle Definitions for USBS X-Y Antenna Mounts

where

A = azimuth angle referred to true north (0 to 360°)

E = elevation angle (0 to 90°)

x and y are 20' antenna site raw data angles

85 ft. USB antenna sites:

$$\sin A = \frac{\sin y}{\cos E} \quad (8)$$

$$\cos A = \frac{-\cos y \sin x}{\cos E} \quad (9)$$

$$\sin E = \cos y \cos x \quad (10)$$

where: A and E are defined previously and x and y are 85' antenna measurements.

The time-tag associated with all USB angle data is ground receive time corrected on-site for WWV propagation delay.

#### 4.4 USB Site Locations

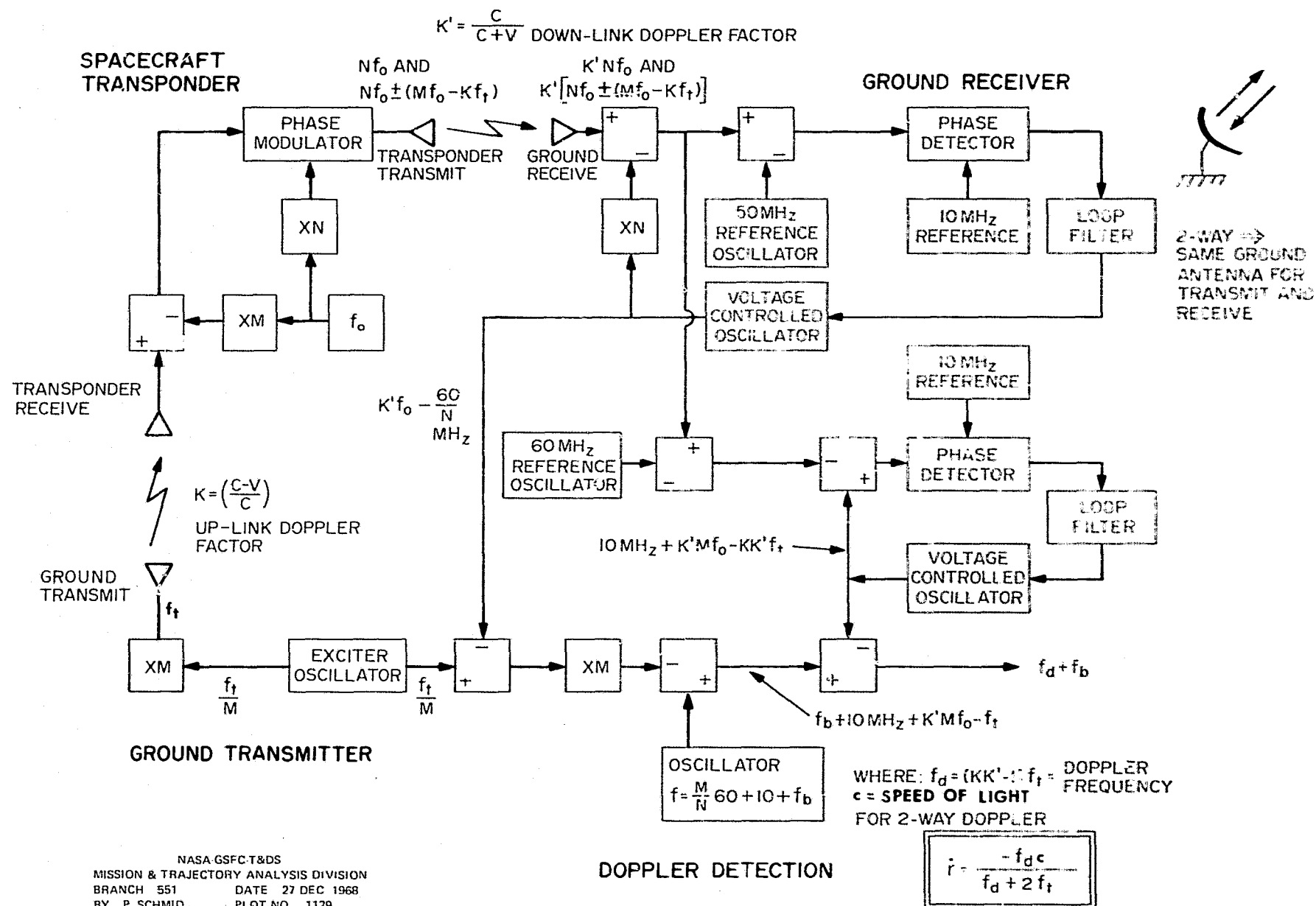
Figure 7 presents USB station locations in geodetic coordinates (ref. 7).

### 5.0 GODDARD RANGE AND RANGE-RATE (GRARR) SYSTEM

The Goddard Range and Range-Rate (GRARR) system (ref. 16, 17, 18) incorporates a non-coherent transponder at the vehicle being tracked. The up-link signal to the spacecraft is translated in frequency at the vehicle transponder by an onboard oscillator and then re-transmitted to the ground receiver. A reference signal related to the on-board translation oscillator is also sent to the ground to permit an appropriate demodulation of the turned around signal. The basic scheme is shown in figure 8 and more detail regarding the ground Doppler processing technique is given in the Appendix.

STATION	STATION NAME	TYPE	LATITUDE (DEGREES)	LONGITUDE (DEGREES)	HEIGHT (METERS)
MIL	MERRITT IS	USBS30	28.508272	-80.693417	10
BDA	BERMUDA	USBS30	32.351286	-64.658181	21
GBM	GRAND BAHAMA	USBS30	26.632858	-78.237664	5
ANG	ANTIGUA IS	USBS30	17.016692	-61.752689	50
CRO	CARNARVON, AUSTRALIA	USBS30	-24.907592	113.724247	58
HAW	HAWAII	USBS30	22.124897	-159.664989	1150
GYM	GUAYMAS, MEXICO	USBS30	27.963206	-110.720850	19
TEX	CORPUS CHRISTI, TEX	USBS30	27.653750	-97.378469	10
GWM	GUAM	USBS30	13.309244	144.734414	127
GDS	GOLDSTONE, CALF.	USBS85	35.341694	-116.873289	965
GLD	PIONEER	USBS85	35.389669	-116.819061	1029
ACN	ASCENSION IS	USBS30	-7.955056	-14.327578	562
HSK	CANBERRA, AUST.	USBS85	-35.597222	148.979167	1097
NBF	TIDBINBILLA, AUST.	USBS85	-35.402233	148.580058	673
MAD	MADRID, SPAIN	USBS85	40.455358	-4.167394	825
RID	CEREBROS, SPAIN	USBS85	40.452983	-4.366767	778
CYI	GRAND CANARY IS	USBS30	27.764536	-15.634536	173

Figure 7—Unified S-Band Site Locations



### Figure 8—Goddard Range and Range Rate S-Band Doppler Tracking

As in the USB system, carrier tracking provides the average range-rate information. By suitably combining the down-link reference and down-link Doppler shifted signal (refer to figure 8) the process is mathematically equivalent to 2-way Doppler processing with a transponder turnaround constant of unity (i.e.  $k = 1$  in eq. 2 for GRARR). Figure 8 is for S-Band operation. VHF Doppler processing is handled in similar fashion.

The range measurement (ref. 19) is obtained by observing the relative phase shift of a low frequency sinusoidal modulation placed upon a subcarrier of the up-link signal. The lowest sidetone frequency is 8 Hz. The range measurement consists of a time delay comparison of the tone modulation sent via the spacecraft transponder and a ground generated reference tone. With the 8 Hz tone the maximum unambiguous range corresponds to the radial distance required to produce an audio phase shift equal to one-half the audio wavelength or

$$\frac{\lambda}{2} = \left( \frac{c}{2f} \right) = \frac{1}{2} \left( \frac{2.997925 \times 10^5 \text{ km/s}}{8 \text{ Hz}} \right) \quad (11)$$

$$= 18737 \text{ km (maximum unambiguous range)}$$

The angle measurements for the Goddard Range and Range-Rate system consist of  $x$  and  $y$  angles with the  $X$  axis aligned North-South just as in the case of the MSFN 30' sites.

### 5.1 GRARR Ranging

The range measurement is based on the measured propagation delay as evidenced by the zero crossings of the phase delayed ranging tone. As usual the assumption is made that signal time delay or phase shift is equal for both ground transmission to spacecraft and spacecraft transmission back to ground. The total two-way time delay is equal to the total number of full cycles the lowest ranging tone is shifted plus the relative incremental phase shift of the tone as measured at the ground receiver. This measured time delay includes, of course, the on-site delays such as cable or waveguide runs as well as the spacecraft transponder time delay. Thus the one-way range measurement is given by:

$$R = \frac{c}{2} \left[ \frac{N}{F_L} + T_R - T_T - T_B \right] \text{ km} \quad (12)$$

where:

$R$  = range measurement

$C$  = speed of light =  $2.997925 \times 10^5$  km/s

$N$  = number of full cycles of phase shift at lowest sidetone frequency  $F_L$ . (Usually  $F_L = 8\text{Hz.}$ )

$T_R$  = incremental time difference as measured at ground receiver

$T_T$  = transponder bias subtracted by METCAF

$T_B$  = on-site biases subtracted by METCAF

The ambiguity represented by  $N/F_L$  is solved for by the METCAF program by means of an integrated trajectory computation. The time tag associated with this range is Universal Time Coordinated (UTC) ground range measurement time corrected for WWV radiowave propagation time delay. The range time tag is presented as day of year, hours, minutes and seconds to the nearest microsecond.

In the GRARR system there is a raw data frame time,  $T_F$ , followed by 4 frames of data at a data rate  $T_{DR}$ . The appropriate time tag for subsequent frames is, therefore, given by:

$$T_S = T_F + kT_{DR} \text{ seconds} \quad (13)$$

where

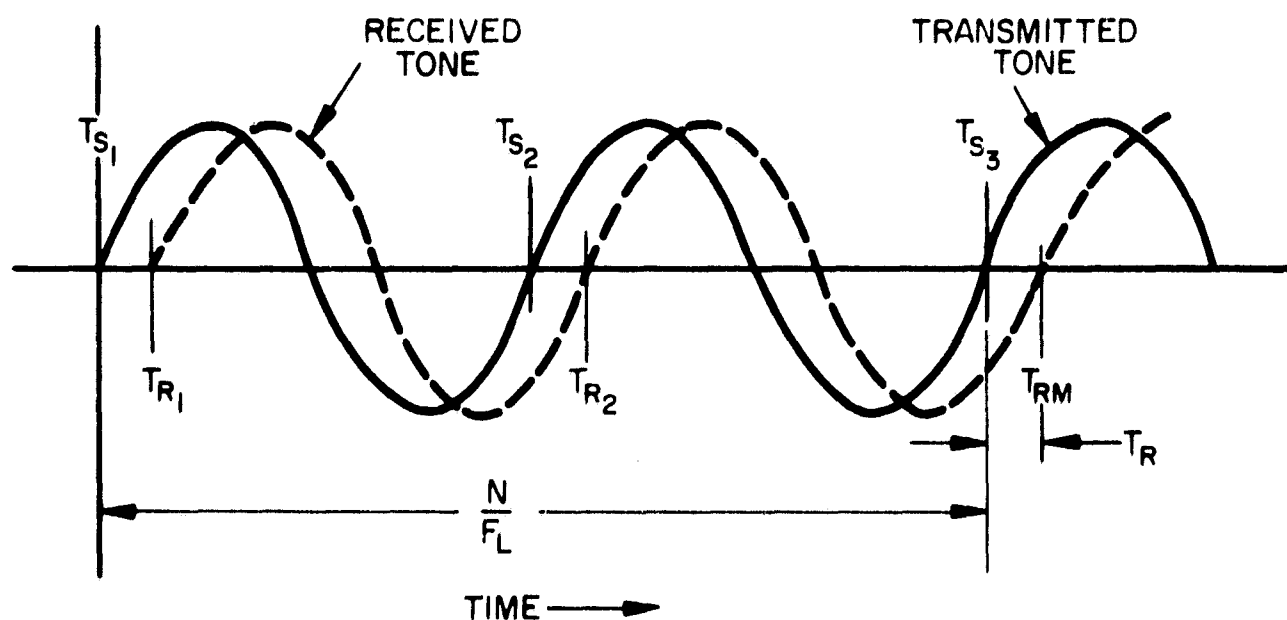
$T_S$  = frame time

$T_F$  = basic frame time

$T_{DR}$  = data rate

$k = 0, 1, 2, 3$

It should be noted that the time tag,  $T_S$ , is associated with the positive going zero crossing of the lowest range tone frequency just prior to an incremental range time delay measurement  $T_R$ . The time relationships are indicated in figure 9. The time tag,  $T_S$ , is "transmit time" in the sense that it corresponds



**NOTE:**

1. TIME UTC IN DAYS, HOURS, MINUTES, SECONDS
2.  $T_S$  = FRAME TIME
3.  $T_{RM}$  = TIME OF RANGE MEASUREMENT
4. 2-WAY DELAY TIME GIVEN BY  $T_R + \frac{N}{F_L}$
5.  $N$  = NUMBER OF FULL CYCLES  
LOWEST RANGE TONE  $F_L$  IS  
SHIFTED DURING 2-WAY  
PROPAGATION.

NASA-GSFC-T & DS  
MISSION & TRAJECTORY ANALYSIS DIVISION  
BRANCH 551  
BY P. SCHMID  
DATE 27 DEC. 1968  
PLOT NO. 1119

Figure 9-Goddard Range and Range-Rate System Time Tagging

to the time that all transmitted range tones pass through a positive going zero crossing. The ground measurement time, which is not influenced by ambiguity resolution, is given by;

$$T_{RM} = T_S + T_R + T_W \quad (14)$$

where

$T_{RM}$  = range time tag (ground measurement time)

$T_S$  = frame time

$T_W$  = inserted WWV time delay correction

$T_R$  = time increment corresponding to relative phase difference between transmitted and received sidetone frequency.

For tracking data recorded after November 1968, WWV correction has been made on-site.

## 5.2 GRARR Range-Rate

The overall electronics of the Goddard Range and Range-Rate system is such that it is equivalent to having a coherent transponder with a turnaround constant  $k = 1$  (see figure 8). Thus, the same two-way average range-rate equation used for the USB is applicable by simply setting  $k = 1$ . Also, it should be noted that the final output of the Doppler processor of the GRARR system is related to a "plus" Doppler shift whereas, as indicated in figure 5, for the USB system it is the "negative" of Doppler shift. At present all GRARR Doppler measurements are N count, destruct. Based on the foregoing:

$$\dot{r}_a = - \left( \frac{\frac{\Delta N}{\Delta T} - f_b}{\frac{\Delta N}{\Delta T} - f_b + 2f_t} \right) c = \frac{-f_d c}{f_d + 2f_t} \text{ km/s} \quad (15)$$

where

$\dot{r}_a$  = average range-rate

$c = 2.997925 \times 10^5 \text{ km/s}$

$f_b$  = inserted bias

$\Delta N$  = a fixed number of Doppler plus bias cycles counted

$\Delta T$  = time required to count  $\Delta N$  cycles (information contained in raw data format in the form of number of cycles of a reference frequency)

$f_t$  = transmitter frequency (both S-Band and VHF are used)

$f_d$  = average Doppler frequency shift.

The "TRADES" time tag associated with the GRARR range rate is UTC ground measurement time corrected for WWV radiowave propagation delay and referenced to the center of the count interval  $\Delta T$ . Note that the Doppler count,  $\Delta T$ , starts at  $T_s$  (figure 9). The range-rate time tag,  $T_{RM}$ , is therefore given by:

$$T_{RM} = T_s + \frac{\Delta T}{2} + T_w \quad (16)$$

where

$T_{RM}$  = range rate time tag

$T_s$  = frame time

$T_w$  = inserted WWV time delay correction

$\Delta T$  = measured Doppler count time interval required to accumulate  $\Delta N$  cycles of carrier phase change.

For tracking data recorded after November 1968, WWV correction has been made on-site. It should also be noted that the data frame time tag  $T_s$  in this case is not in "real time." That is, the time,  $\Delta T$ , it takes to count  $N$  cycles of Doppler is, of course, not known at the beginning of the count interval and the frame time record must be delayed appropriately until the Doppler time interval is measured. This delay in time tagging is done at the STADAN site such that no fixed time tag bias is introduced. This is in contrast to the MSFN destruct Doppler time tagging indicated in section 4.2 which must be corrected by the raw data user to reflect the single frame time delay. Also by mid-year 1969, all GRARR stations except Santiago (Chile) will have been modified such that equation (15) is independent of transmit frequency,  $f_t$ . This will be achieved by generating the reference frequency (used to measure  $\Delta T$ ) and the bias frequency,  $f_b$ , as linear functions of  $f_t$  (ref. 4).

### 5.3 GRARR Angles

The basic measurement is  $x$  and  $y$  angles. The  $X$  axis is aligned North-South and the azimuth and elevation equations 5, 6, and 7 which apply to the 30 f MSFN sites also apply to the GRARR sites. The GRARR angles are time tagged at ground receive time.

That is:

$$T_a = T_F + T_W \text{ seconds} \quad (17)$$

where

$T_F$  = basic frame time

$T_W$  = WWV propagation correction

$T_a$  = angle time tag

Note that only one angle read-out is available in the raw data for every four frames of range and range rate data. For tracking data recorded after November 1968, WWV correction has been made on-site.

### 5.4 GRARR Site Locations

Figure 10 presents STADAN station locations (both VHF and S-Band GRARR sites) in geodetic coordinates (ref. 20).

## 6.0 C-BAND RADAR

The FPQ-6 and FPS-16 pulse type radar systems provide two basic measurements; namely, range and angles. The angles recorded are azimuth and elevation.

### 6.1 C-Band Ranging

The range measurements obtained from a beacon track are corrected for transponder time delays on-site. System time delays are also subtracted on-site. The raw data is in yards and is converted to km using 1 meter = 3.280839895 international ft. The time tag at the METCAF program output is receive time and is corrected on-site for WWV propagation time delay.

STATION	STATION NAME	TYPE	LATITUDE (DEGREES)	LONGITUDE (DEGREES)	HEIGHT (METERS)
TAN	TANANARIVE	SBAND	-19.019844	47.302625	1385
TAN	TANANARIVE	VHF	-19.020939	47.302625	1385
ROS	ROSMAN, NC	SBAND	35.196034	-82.875864	876
ROS	ROSMAN, NC	VHF	35.194934	-82.875864	876
SAN	SANTIAGO, CHILE	SBAND	-33.156481	-70.666742	695
SAN	SANTIAGO, CHILE	VHF	-33.151167	-70.666742	695
FRB	FAIRBANKS, ALASKA	SBAND	64.972167	-147.512389	371
FRB	FAIRBANKS, ALASKA	VHF	64.971697	-147.510803	371
CRO	CARNARVON	SBAND	-24.904128	113.715294	51
CRO	CARNARVON	VHF	-24.905228	113.715294	51

Figure 10--STADAN Goddard Range and Range-Rate Site Locations

## 6.2 C-Band Angles

The METCAF output is raw azimuth and elevation data converted from mils to degrees. The conversion is based on 6400 mils (exact) being equivalent to  $360^\circ$ . The time tag is again ground receive time corrected on-site for WWV propagation time delay.

## 6.3 C-Band Site Locations

Figure 11 presents the C-Band radar site locations of stations most often used in NASA satellite tracking. The locations are in geodetic coordinates (ref. 7).

## 7.0 SUMMARY

The purpose of the "TRADES" (Tracking Data Editing System) computer programming effort is to permit a straightforward conversion of STADAN and MSFN on-site recorded tracking data to metric form. Figure 1 presented previously indicates the overall data conversion scheme effected by the TRADES program.

The metric information contained in the TRADES program output format is summarized in Tables II through IV. All time tags are at ground measurement time and are corrected for WWV propagation delay. All range measurements include corrections for transponder and station time delays where appropriate.

All angle data is presented in two forms, namely, azimuth and elevation and x and y angles. The C-Band azimuth and elevation are also presented as X-Y angle data with the X-axis aligned North and South. This is the same X-axis orientation as the 30' USB sites and GRARR sites. The 85' USB sites have an East-West X-Y orientation.

The range-rate information presented in the output necessarily represents an "average range rate" which, as explained in the Appendix, must in general be corrected by the orbit computation user to reflect "true range rate." However, sufficient information is presented in the TRADES program output format to convert this measurement to a radial range change (spacecraft relative to tracking site) which, except for system errors, represents an exact measure of spacecraft net radial motion over the measurement interval. Thus orbit and trajectory determinations based upon range changes can also be implemented.

STATION	STATION NAME	TYPE	LATITUDE (DEGREES)	LONGITUDE (DEGREES)	HEIGHT (METERS)
BDA	BERMUDA	FPQ-6	32.347964	-64.653742	19
BDA	BERMUDA	FPS-16	32.348103	-64.653800	18
CYI	GRAND CANARY IS	MPS-26	27.763206	-15.634814	168
CRO	CARNARVON, AUSTRALIA	FPQ-6	-24.897378	113.716078	62
WOM	WOOMERA, AUSTRALIA	FPS-16	-30.819728	136.836989	151
HAW	HAWAII	FPS-16	22.122092	-159.665383	1140
CAL	PT. ARGUELLO, CALF.	FPS-16	34.582903	-120.561150	646
EGL	EGLIN AF BASE	FPS-16	30.421767	-86.798114	28
PAF	PATRICK AF BASE	FPQ-6	28.226553	-80.599292	15
CNV	CAPE KENNEDY	FPS-16	28.481767	-80.576514	14
GBI	GRAND BAHAMA	FPS-16	26.615778	-78.347833	14
GBI	GRAND BAHAMA	TPQ-18	26.636350	-78.267722	12
GTI	GRAND TURK IS	TPQ-18	21.462889	-71.132114	28
SSI	SAN SALVADOR IS	FPS-16	24.118825	-74.504136	5
MLA	MERRITT IS	TPQ-18	28.424860	-80.664417	12
ASC	ASCENSION IS	FPS-16	-7.934847	-14.412606	110
ASC	ASCENSION IS	TPQ-18	-7.972761	-14.401694	143
PRE	PRETORIA, S. AFRICA	MPS-25	-25.943734	28.358489	1626
WHS	WHITE SANDS, N.M.	FPS-16	32.358222	-106.369564	1232
ANT	ANTIGUA IS	FPQ-6	17.144031	-61.792858	58

Figure 11-C-Band Site Locations

Table II  
Range Output

System	Range (km)	Time Tag (Day, Hr, Min, Sec)
USB	km to nearest mm	Ground receive time to nearest microsecond
GRARR	km to nearest mm	Ground receive time to nearest microsecond
C-Band	km to nearest mm	Ground receive time to nearest microsecond

Table III  
Average Range-Rate Output

System	Range-Rate (km/sec)	Time Tag (Day, Hr, Min, Sec)
USB	km/sec to nearest 0.1 mm/sec	To center of measurement interval ground receive time to nearest microsecond
GRARR	km/sec to nearest 0.1 mm/sec	To center of measurement interval ground receive time to nearest microsecond
C-Band	Not Applicable	Not Applicable

Table IV  
Angle Output

System	Angle (degrees)			Time Tag (Day, Hr, Min, Sec)
	X	Y	AZ-EL	
30' USB	X-Axis aligned North-South 0 to +90° toward East 0 to -90° toward West	0 to +90° toward North 0 to -90° toward South	To nearest 0.001°	Ground receive time to nearest microsecond
85' USB	X-Axis aligned East-West 0 to +90° toward South 0 to -90° toward North (All Angles to Nearest 0.001°)	0 to +90° toward East 0 to -90° toward West		Ground receive time to nearest microsecond
GRARR	(Same as 30' USB Site)			Ground receive time to nearest microsecond — one sample every 4 data frames
C-Band	(Same as 30' USB Site)		↓	Ground receive time to nearest microsecond

NOTES: (Tables II through IV)

1. Time tags corrected for WWV propagation time delay.
2. Range measurements include transponder and station delays.
3. Azimuth referred to North 0 to 360° positive angles toward East.
4. Elevation 0 to 90°, zenith = 90°.
5. Raw angle measurements: C-Band AZ-EL; GRARR and USB X-Y.

Once the on-site tracking data is converted to metric form, meaningful post-flight analysis is possible prior to orbit and trajectory computation. The purpose of such analysis is to provide an evaluation of tracking system performance during an actual tracking exercise. Examples of information being provided by such analyses include;

1. antenna angle servo closed loop noise equivalent bandwidth (ref. 2)
2. maximum sample rates corresponding to uncorrelated data samples.
3. variation in short term fluctuations of the metric data attributable to radiowave propagation through the earth's atmosphere as a function of elevation angle of observation (ref. 21).
4. range-rate cyclic variations associated with satellite spins (ref. 22).
5. evaluation of automated as well as operator inserted data quality and status indicators.
6. cyclic variations introduced by range tracking schemes such as employed in the FPS-16 and FPQ-6 pulse radar systems (ref. 5).

Information of this nature is often more easily identified prior to the filtering of an orbit computation program.

Then too, data converted to metric form is suitable as input to orbit and trajectory analysis programs. The NAP I (Network Analysis Program) and NAP II programs of the Mission and Trajectory Analysis Division use the standard output of the TRADES program as a basic input. Orbit and trajectory analysis leads to refinements in station location, gravitational models, timing schemes and determination of biases such as introduced by the Earth's troposphere and ionosphere.

## APPENDIX

### THE INTERPRETATION OF DOPPLER MEASUREMENTS

#### Derivation of Doppler Equation:

The following heuristic derivation for the USB 2- and 3-way range-rate measurement is based upon the development of Dr. B. Kruger of MTAD (ref. 23). The derivation assumes a coherent frequency turnaround where the transponder transmit carrier frequency is a rational fraction,  $k$ , times the transponder received signal. As indicated previously the Goddard Range and Range-Rate system, although not employing a coherent transponder, achieves the same result by means of suitable ground receiver processing, as employing a coherent turnaround constant  $k$  equal to unity. It is interesting to note that the result derived by what might be considered classical means is for the case of 2-way Doppler, exactly what one obtains by means of successive applications of the Lorentz transformation, a result which is rigorous for the case of Special Relativity.

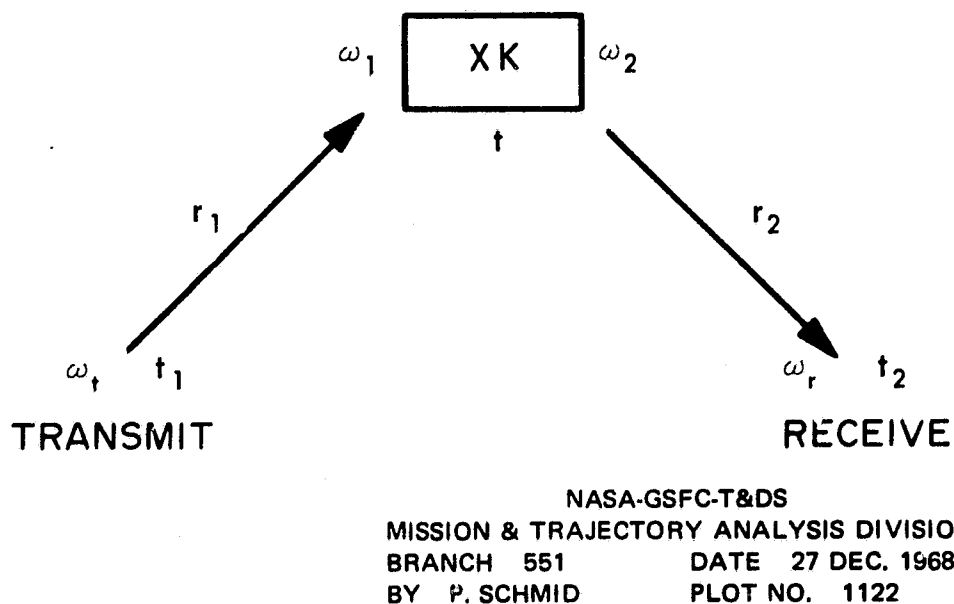


Figure 12-Coherent Doppler Tracking

The following definitions apply to figure 12.

$t_1$  = the ground transmit time a given segment of a sinusoidal continuous wave (cw) radiofrequency signal is transmitted (seconds)

$\omega_t$  = angular frequency of transmitted signal (radians/second)

$r_1$  = radial distance from transmitting antenna to vehicle being tracked (meters)

$\omega_1$  = the angular frequency of the Doppler shifted uplink signal as received by the vehicle transponder

$t$  = vehicle time corresponding to the time the given transmitted segment is received at the vehicle

$k$  = the coherent multiplication factor at transponder = 240/221 for the USB system

$\omega_2 = k\omega_1$  the transmitted frequency at time  $t$  (radians/second)

$t_2$  = ground receive time of the reference segment which was transmitted at time  $t_1$  (seconds)

$\omega_r$  = angular frequency of received signal which is a function of the transmitter frequency  $\omega_t$ , the uplink Doppler shift, the turnaround constant  $k$  and the downlink Doppler shift

$r_2$  = radial range between vehicle and receive antenna.

NOTE:

1. for two-way Doppler tracking the same antenna is used for transmit and receive and  $r_1 = r_2 = r$
2. for three-way Doppler tracking  $r_1$  and  $r_2$  are functions of respective ground antenna locations
3. the turnaround time at the transponder is assumed negligible

With these definitions the Doppler shifted received signal,  $\omega_r$ , is determined and the angular Doppler shift  $\omega_d$  can be related to the respective range rates  $\dot{r}_1$  and  $\dot{r}_2$ . One starts with the definition of angular frequency:

$$\omega = 2\pi f = \frac{d\phi}{dt}$$

where

$f$  = frequency (Hz)

$\frac{d\phi}{dt}$  = time rate of change of phase (radians/sec).

The entire derivation amounts to the tracing through of a small change of phase,  $d\phi$ , as generated at the transmitter and keeping track of the corresponding times  $t_1$ ,  $t$ , and  $t_2$ . Thus the frequencies  $\omega_r$ ,  $\omega_1$ ,  $\omega_2$  and  $\omega_t$  become:

$$\omega_t = \frac{d\phi}{dt_1}$$

$$\omega_1 = \frac{d\phi}{dt}$$

$$\omega_2 = \frac{k d\phi}{dt}$$

$$\omega_r = \frac{k d\phi}{dt_2}$$

or

$$\frac{\omega_t}{\omega_r} = \frac{dt_2}{k dt_1} \quad (\text{A-1})$$

It is noted that:

$$t = t_1 + \frac{r_1(t)}{c} \quad (\text{A-2})$$

and

$$t_2 = t_1 + \frac{r_1(t)}{c} + \frac{r_2(t)}{c} \quad (\text{A-3})$$

where  $r/c$  corresponds to the propagation time delay in freespace in which case  $c$  corresponds to the speed of light. If the velocity of propagation is other than  $c$  then appropriate corrections must be applied. For example, at frequencies below 1 GHz both the effects of the earth's ionosphere and troposphere must be considered. Above approximately 1 GHz only the troposphere need be considered when propagating through the earth's atmosphere.

However, for purposes of this development, the phase velocity of the electromagnetic energy is taken to be the free space value. Equation (A-2) is differentiated with respect to vehicle time  $t$  and becomes

$$1 = \frac{dt_1}{dt} + \frac{dr_1}{cdt} = \frac{dt_1}{dt} + \frac{\dot{r}_1}{c} \quad (\text{A-4})$$

where  $\dot{r}_1$  is a range rate as measured in vehicle time  $t$  (m/sec) equation (A-4) can be rewritten as

$$\frac{dt}{dt_1} = \frac{1}{1 - \frac{\dot{r}_1}{c}} \quad (\text{A-5})$$

similarly equation (A-3) is differentiated with respect to transmit time,  $t_1$ , to obtain:

$$\frac{dt_2}{dt_1} = 1 + \frac{dr_1}{cdt} \frac{dt}{dt_1} + \frac{dr_2}{cdt} \frac{dt}{dt_1}$$

$$\text{or} \quad \frac{dt_2}{dt_1} = 1 + \frac{\dot{r}_1}{c} \frac{dt}{dt_1} + \frac{\dot{r}_2}{c} \frac{dt}{dt_1} \quad (\text{A-6})$$

Combining (A-5) and (A-6)

$$\frac{dt_2}{dt_1} = 1 + \frac{\dot{r}_1}{c} \frac{1}{\left(1 - \frac{\dot{r}_1}{c}\right)} + \frac{\dot{r}_2}{c} \frac{1}{\left(1 - \frac{\dot{r}_1}{c}\right)} \quad (\text{A-7})$$

Comparison of (A-7) and (A-1) reveals that  $\omega_t/\omega_r$  is given by

$$\frac{\omega_t}{\omega_r} = \frac{dt_2}{k dt_1} = \frac{1}{k} \left[ 1 + \frac{\dot{r}_1}{c} \frac{1}{\left(1 - \frac{\dot{r}_1}{c}\right)} + \frac{\dot{r}_2}{c} \frac{1}{\left(1 - \frac{\dot{r}_1}{c}\right)} \right] \quad (\text{A-8})$$

Equation (A-8) can be rewritten as;

$$k\omega_t = \omega_r \frac{\left(1 + \frac{\dot{r}_2}{c}\right)}{\left(1 - \frac{\dot{r}_1}{c}\right)} \quad (\text{A-9})$$

The Doppler shift at the receiver is defined as the difference between the received angular frequency  $\omega_r$  and the signal which would be received in the absence of spacecraft radial motion relative to the tracking site (or sites in the case of 3-way Doppler tracking), that is;

$$\omega_d = \omega_r - k\omega_t \quad \text{radians/sec} \quad (\omega = 2\pi f)$$

$$\text{or} \quad f_d = f_r - kf_t \quad \text{Hz} \quad (\text{A-10})$$

Combining (A-9) and (A-10)

$$kf_t = [f_d + kf_t] \left[ \frac{1 + \frac{\dot{r}_2}{c}}{1 - \frac{\dot{r}_1}{c}} \right] \quad (\text{A-11})$$

It is noted that equation (A-11) completely describes the rates of radial range change in vehicle time ( $\dot{r}_1$  and  $\dot{r}_2$ ) in terms of the measured total Doppler frequency shift,  $f_d$ , the accurately maintained transmit frequency,  $f_t$ , and the turnaround constant  $k$ . In the case of 2-way Doppler where the receive and transmit antennas are collocated  $r_1 = r_2 = r$  and equation (A-11) becomes:

$$\dot{r} = \frac{-f_d c}{f_d + 2kf_t} \quad (\text{A-12})$$

Equation (A-12) is as indicated in figure 5 where it is noted that the USB system actually records a minus Doppler shift plus 1 MHz bias. So for the USB system (2-way Doppler tracking) equation (A-12) becomes

$$\dot{r}_a = \frac{\Delta R}{T} = \frac{-f_d c}{f_d + 2kf_t} = \frac{\left(\frac{\Delta N}{T} - f_B\right) c}{-\left(\frac{\Delta N}{T} - f_B\right) + 2kf_t} \text{ m/s} \quad (\text{A-13})$$

which corresponds to equation (2) of section 4.2.

### The Relationship Between Actual and Average Range-Rate:

The quantity measured in 2-way Doppler tracking is actually change in radial range rather than radial velocity. Thus in equation (A-13)  $\dot{r}_a$  is an average range-rate which is generally time tagged by orbit computation schemes as vehicle time at the mid-point of the measurement interval T. As will be shown the selection of this midpoint time tag minimizes the complexity of the correction scheme required to ascertain true radial range,  $\dot{r}$ , from the average range-rate,  $\dot{r}_a$  (ref. 23).

With reference to figure 13, the basic measurement achieved by Doppler tracking is that of a change in radial range (sometimes termed slant range),  $\Delta R$ . This change, in the case of 2-way carrier Doppler tracking is evidenced by a cycle of Doppler (or one Doppler "count") each time the vehicle has moved approximately  $\lambda/2$  of the up-link wavelength. At 2 GHz this amounts to a Doppler count each time the vehicle has moved radially 7.5 cm relative to the tracking station. This is seen from equation (A-13) where:

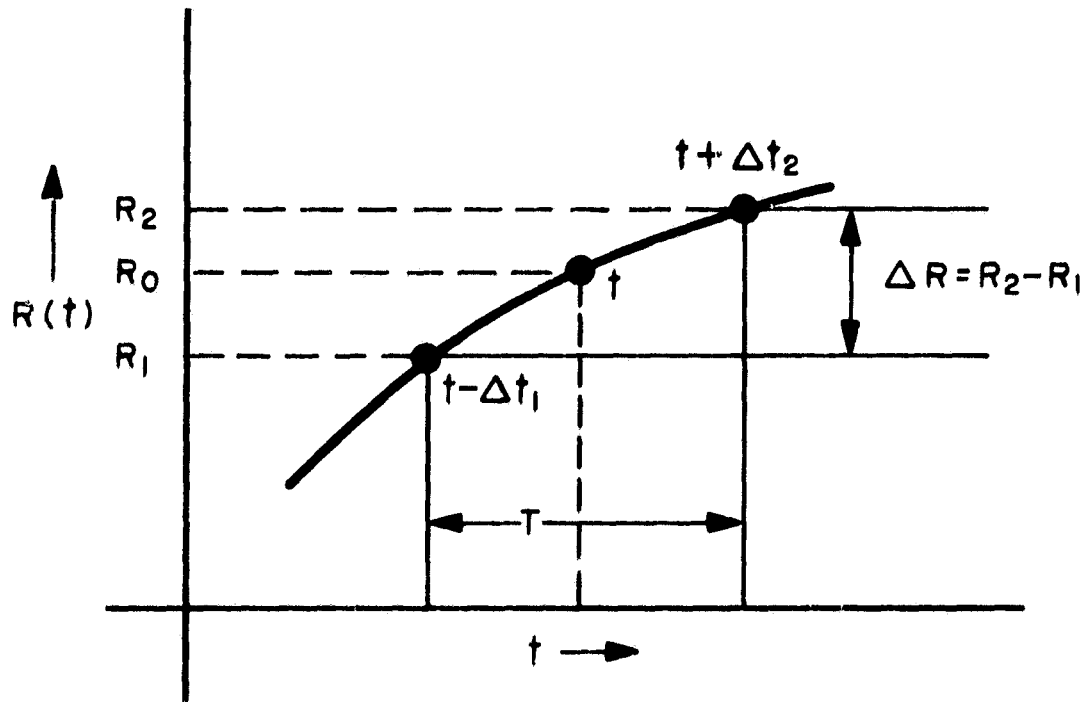
$$\dot{r}_a = \frac{\Delta R}{T} = \frac{-f_d c}{f_d + 2kf_t} = -\frac{\frac{n}{T} c}{2kf_t}$$

(where  $n$  = cycles of Doppler counted)

and for  $k \doteq 1$

$$\Delta R \doteq \frac{-nc}{2f_t} = -n \left( \frac{\lambda}{2} \right) \quad (A-14)$$

If  $n$  is recorded by counting positive zero crossings for example, the smallest measurement possible or granularity is  $\lambda/2$  at the carrier frequency, where  $\lambda$  corresponds to the carrier. If a modulation scheme is employed, where the phase shift of a clock or low frequency tone is recorded then the clock or tone Doppler is also given by (A-14) providing  $\lambda$  is associated with the modulating frequency.



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Figure 13--Radial Range Change as a Function of Time

Again with reference to figure 13 the change in radial range,  $\Delta R$ , can be expressed in terms of a value of slant range,  $R_0$ , somewhere in interval  $T$  and series expansions about  $R_0$ . That is:

$$\begin{aligned} \Delta R = R_2 - R_1 = & \left[ R_0 + \dot{R}_0 \Delta t_2 + \ddot{R}_0 \frac{\Delta t_2^2}{2} + \frac{\ddot{\ddot{R}}_0}{6} \Delta t_2^3 + \dots \right] \\ & - \left[ R_0 - \dot{R}_0 \Delta t_1 + \frac{\ddot{R}_0}{2} \Delta t_1^2 - \frac{\ddot{\ddot{R}}_0}{6} \Delta t_1^3 + \dots \right] \end{aligned} \quad (A-15)$$

Now it is noted that if the time tag is selected at the midpoint of the sample time,  $T$ , then

$$\Delta t_1 = \Delta t_2 = \frac{T}{2}$$

and all even derivative terms drop out such that:

$$\Delta R = \dot{R}_0 T + \frac{\ddot{R}_0 T^3}{24} + \frac{\dddot{R}_0 T^5}{1920} + \dots \quad (A-16)$$

and the "true range-rate,"  $\dot{R}_0$ , as evaluated in vehicle time at the center of the time arc  $T$  becomes:

$$\dot{R}_0 = \frac{\Delta R}{T} - \frac{\ddot{R}_0 T^2}{24} - \frac{\dddot{R}_0 T^4}{1920} \quad (A-17)$$

$\frac{\Delta R}{T}$  = average range rate as determined by tracking equipment (see for example equation (A-13))

$T$  = sample time (assumed constant for all practical purposes during time of correction) for USB non-destruct  $T$  count Doppler  $T$  = sample rate = a constant except for data frames edited out.

$\ddot{R}_0$  and  $\dddot{R}_0$  are higher order derivatives which must be estimated. This is usually obtained via an orbit determination ephemeris, however, as shown in ref. 24, meaningful correction terms can be derived directly from the raw tracking data. It should also be noted that selection of the midpoint of the Doppler count minimizes the correction term associated with  $\ddot{R}_0$ . That is the  $\ddot{R}_0$  (equation A-15) coefficient is proportional to

$$\Delta t_1^3 + \Delta t_2^3 = \Delta t_1^3 + (T - \Delta t_1)^3 = \Phi$$

or

$$\Phi = T^3 - 3\Delta t_1 T^2 + 3T\Delta t_1^2$$

Let

$$\left[ \frac{d\Phi}{d\Delta t_1} \right] = 0 = 6T\Delta t_1 - 3T^2$$

or

$$\Delta t_1 = \frac{T}{2}$$

to assure a minimum evaluate

$$\frac{d^2 \Phi}{d\Delta t_1^2} = 6T > 0$$

= > a minimum

Hence the selection of the mid-point minimizes the correction term associated with  $\ddot{R}_0$  (equation A-17).

#### Non-Coherent Transponder Turnaround:

The Goddard Range and Range-Rate system does not employ a coherent transponder at the spacecraft being tracked in the 2-way Doppler mode. That is, the downlink signal, from the spacecraft, is not phase related to the uplink signal from the ground transmitter. However, the uplink signal is frequency translated by means of an onboard oscillator at the transponder and a reference signal, phase related to this translation oscillator, is transmitted along with the turned around signal to the ground receiver.

The ground Doppler receiver configuration is as previously indicated in figure 8. It will be noted that the final output to the Doppler counter is the inserted bias frequency which permits direction of radial velocity determination, plus the Doppler frequency. Thus it is seen that by suitable manipulation of the received signal, the Goddard Range and Range-Rate system produces an output which corresponds to what would be obtained with a coherent turnaround at the transponder with a frequency multiplication constant,  $k$ , of unity. For practical reasons, a coherent frequency turnaround with  $k = 1$  is seldom employed in a transponder since the isolation between transponder transmitter and receiver must, in a continuous wave system, be achieved by means of separate antennas. Such isolation at any given frequency cannot be expected to exceed 80 db a level not generally acceptable in light of receiver sensitivities on the order of -120 dBm and spacecraft transmitter power levels well in excess of one milliwatt, (ref. 25).

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## ABBREVIATIONS USED IN THIS REPORT

C-Band	Nominal frequency range of 4 to 8 GHz (C-Band pulse radar equipment operates from 5.4 to 5.9 GHz)
GHz	$10^9$ Hz
GSFC	Goddard Space Flight Center, Greenbelt, Maryland
GRARR	Goddard Range and Range-Rate Tracking System
Hz	Unit of frequency, 1 Hertz = 1 cycle per second
METCAF	Metric Calibration and Formatting Program (reformats USB data)
MHz	$10^6$ Hz
MSFN	Manned Space Flight Network
MTAD	Mission and Trajectory Analysis Division, GSFC
NASCOM	NASA Communications Network
S-Band	Nominal frequency range of 2 to 4 GHz
STADAN	Space Tracking and Data Acquisition Network (STADAN program reformats GRARR data)
TRADES	Tracking Data Editing System program
USB	Unified S-Band system
UTC	Universal Time Coordinated is referenced to atomic time (ref. 11 provides an excellent description of the UTO, UT1, UT2 and UTC time scales)
VCO	Voltage Controlled Oscillator
VHF	Frequency range of 30 to 300 MHz (GRARR VHF 136-148 MHz)